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NUCLEAR SAFETY

LIMIT LINES FOR RISK

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Approaches to the regulation of risk from technological systems, such as nuclear power plants or chemical process plants, in which potential accidents may result in a broad range of adverse consequences must take into account several different aspects of risk. These include overall or average risk, accidents posing high relative risks, the rate at which accident probability decreases with increasing accident consequences, and the impact of high frequency, low consequence accidents. A hypothetical complementary cumulative distribution function (CCDF), with appropriately chosen parametric form, meets all these requirements. The Farmer limit line, by contrast, places limits on the risks due to individual accident sequences, and cannot adequately account for overall risk. This reduces its usefulness as a regulatory tool. In practice, the CCDF is used in the Canadian nuclear licensing process, while the Farmer limit line approach, supplemented by separate qualitative limits on overall risk, is employed in the United Kingdom.

I. INTRODUCTION

The purpose of this paper is to examine limit-line approaches to the regulation of risk from technological systems, such as nuclear power plants or chemical process plants, in which potential accidents may result in a broad range of adverse consequences. The variety of possible accident consequences in such cases introduces new elements into the measurement of risk. It is generally recognized, for example, that "average risk" alone is not an adequate measure for nuclear power plants since the potential for very low probability, catastrophic events is a principal concern. Limit-line approaches attempt to control risks from all parts of the spectrum of possible consequences by placing limits on the probabilities of suitably chosen consequence categories or, sometimes, individual accident sequences.

In Sec. II, we develop criteria for limit lines and propose a hypothetical complementary cumulative distribution function (CCDF) as most suitable. Preliminary parameters for such CCDFs are developed. Section III treats the Farmer limit line, the earliest approach to this question, and indicates why we consider it less suitable than the CCDF. We also discuss a controversy surrounding the Farmer line. This illustrates some of the potential for confusion in this approach. In Sec. IV, we examine proposed Canadian criteria for licensing nuclear power plants, as well as actual regulations in force in the United Kingdom. The approaches adopted correspond, respectively, to the approach advocated in this paper and to the Farmer limit line criterion. We present our conclusions in Sec. V, while the Appendix covers the probability theory used in the paper.

II. PROPOSED LIMIT-LINE APPROACH

In controlling or limiting risk from a technological system, many different aspects of risk must be taken into account. They include

1. overall or total risk from the system
2. accidents or categories of accidents posing unusually high relative risks
3. the rate at which accident probability decreases with increasing accident consequences
4. high frequency, low consequence accidents.

Overall or total risk is generally measured by expected or average accident consequences on an

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annual basis,

$$\bar{R} = \sum_i P_i C_i = \int_0^\infty Cf(C)dC \ , \qquad (1)$$

where the sum (or integral, as the case may be) is over all possible accident consequences; see the Appendix for the notation used. The measure $\bar{R}$ does not, however, address the other aspects of risk listed above. Information on items 2, 3, and 4 is lost in the averaging process used to calculate $\bar{R}$. Accidents with high relative risks represent imbalances in the safety of the system and are priorities for risk reduction efforts. Item 3 is concerned with risk aversion, by which we mean the greater importance attached in the public mind to single catastrophic accidents, as opposed to large numbers of minor accidents with similar total consequences. For example, a hazardous activity resulting once a year in a single accident killing 100 people may easily be perceived by the public as more threatening than one resulting in 100 accidents per year, each killing a single individual. A risk averse attitude can be accounted for by requiring that accident probabilities decrease faster than accident consequences increase. It should be noted here that the term "risk aversion" is used in a different sense in decision theory. See Ref. 1 for a comparison of the two types of risk aversion. Finally, concerning item 4, it is important to limit the frequency of low consequence accidents, such as small releases of toxic material or minor injuries to plant personnel, because the cumulative costs of large numbers of such incidents can be great.

All the important aspects of risk may be addressed by using a hypothetical CCDF (see Appendix) as a standard for regulating risk. First, consider the rate at which probability decreases with increasing consequences. As indicated above, we require that probability decrease faster than consequences increase. One way to ensure this is to impose the following condition:

$$Cf(C) \text{ is a decreasing function of } C \ , \quad \text{for } C \geqslant C_0 \ . \qquad (2)$$

Here, f is the probability density function (PDF) of consequences on an annual basis. The restriction $C \geqslant C_0$ is present both for practical and mathematical reasons; the range $C < C_0$ of low consequence accidents is treated separately below. The requirement (2) is equivalent to weak risk aversion as defined in Ref. 1,[a] and can also be rephrased as:

$$\int_C^{C+L} xf(x)dx \text{ is decreasing in } C \ ,$$

$$\text{for } C \geqslant C_0 \ , \quad \text{for each fixed } L > 0 \ . \qquad (3)$$

Requirement (3) holds if and only if

$$\frac{d}{dC}\int_C^{C+L} xf(x)dx = (C+L)f(C+L) - Cf(C) < 0 \qquad (4)$$

for all $C \geqslant C_0$, $L > 0$. Since $L > 0$ is arbitrary, Eq. (4) is equivalent to condition (2). Notice that requirement (3) means that the expected annual consequences for all accidents with consequences in the linear range $[C, C+L]$, of constant length L, is decreasing as a function of C. For example, if consequences are measured in deaths, then accidents causing between 0 and 1000 deaths have greater expected annual consequences than those causing between 1000 and 2000 deaths. This clearly expresses a form of risk aversion. In many situations of interest, however, both probabilities and consequences of potential accidents differ by orders of magnitude so that it is appropriate to use logarithmic scales for probabilities and consequences. In such cases, the form of risk aversion expressed by requirement (3) may not be strong enough. For example, again taking consequences as measured in deaths, and following condition (3), the expected annual number of deaths for accidents causing between 1000 and 10 000 deaths could be far greater than that for those in the range of 1 to 10 deaths. We prefer, therefore, to impose a more stringent risk aversion criterion, namely, that the expected annual consequences for the logarithmic interval $[C, AC]$ be a decreasing function of C. Precisely, we require

$$\int_C^{AC} xf(x)dx \text{ is decreasing in } C \geqslant C_0 \ ,$$

$$\text{for each fixed } A > 1 \ . \qquad (5)$$

This condition is equivalent to

$$\frac{d}{dC}\int_C^{AC} xf(x)dx = \frac{(AC)^2 f(AC) - C^2 f(C)}{C} < 0 \qquad (6)$$

for $C \geqslant C_0$, all $A > 1$. Since A is otherwise arbitrary, Eq. (6) is equivalent to

$$C^2 f(C) \text{ is decreasing for } C \geqslant C_0 \ . \qquad (7)$$

It can be shown that requirement (7) implies (2)[b] so that condition (5) is indeed a stronger requirement

[a]Weak risk aversion (Ref. 1) is expressed by the equation $r(x)f(x)$ = constant, where $r(x)$ is an "impact function" for which $r(x)/x$ is increasing. This implies that $xf(x)$ is decreasing.

[b]We have

$$(d/dC)C^2 f(C) = C^2 (d/dC)f(C) + 2Cf(C) \geqslant C(d/dC)Cf(C) \ ;$$

thus, $(d/dC)C^2 f(C) < 0$ implies that $(d/dC)Cf(C) < 0$.

than condition (3). Condition (5) is not directly comparable to strong risk aversion as defined in Ref. 1. It is, however, closely related to the risk aversion criterion[c] of Kinchin.[2,3]

Let us now specialize to the case where $f(x)$ is proportional to a power of x:

$$f(x) = Kx^{-\alpha} \ , \quad x \geq C_0 \ . \tag{8}$$

Clearly, condition (7) holds if and only if $\alpha > 2$. Thus, the power PDF $f(x) = Kx^{-\alpha}$ gives an adequate *rate of decrease* of probability provided $\alpha > 2$.

Next, consider the range $C \leq C_0$ of low consequence accidents. A possible criterion is that the average annual consequences (i.e., the risk) of such events should not exceed a tolerance level R_1. Finally, let us place a limit $R_2 > R_1$ on the risk due to the entire system. The three requirements above can be met, for example, by a composite PDF of the form

$$\begin{aligned} f(C) &= K_1 \ , \quad \text{if } 0 \leq C < C_0 \\ &K_2 C^{-\alpha} \ , \quad \text{if } C \geq C_0 \ . \end{aligned} \tag{9}$$

The positive constants α, K_1, and K_2 must be chosen so that the following conditions are satisfied:

$$\int_0^\infty f(C)dC = 1 \ ,$$

$$\int_0^{C_0} Cf(C)dC \leq R_1 \ ,$$

and

$$\int_0^\infty Cf(C)dC \leq R_2 \ . \tag{10}$$

These relations lead to:

$$K_2 = (\alpha - 1)(1 - K_1C_0)C_0^{\alpha-1} \ ,$$

$$K_1 \leq \frac{2R_1}{C_0^2} \ ,$$

and

$$\alpha \geq \max\left(2, \frac{2R_2 - C_0}{R_2 - C_0 + \frac{1}{2}K_1C_0^2}\right) . \tag{11}$$

[c]Kinchin requires, in essence, that

$$C\bar{F}(C) = C\int_C^\infty f(x)dx$$

be constant. If, on the other hand, condition (5) holds, we have

$$\frac{d}{dC}C\bar{F}(C) = -Cf(C) + \int_C^\infty f(x)dx < -Cf(C)$$

$$+ C^2 f(C)\int_C^\infty \frac{dx}{x^2} = 0 \ ,$$

for $C \geq C_0$, by condition (7). Thus, condition (5) is somewhat more stringent than Kinchin's criterion.

The conditions on α and K_1 can be illustrated graphically. Figure 1 shows the case $C_0 \leq R_2$: the permissible values of the pair (α, K_1) lie in the shaded region.

The CCDF corresponding to Eq. (9) is given by

$$\begin{aligned} \bar{F}(C) &= 1 - K_1C \ , && \text{if } 0 \leq C < C_0 \\ &= (1 - K_1C_0)\left(\frac{C}{C_0}\right)^{1-\alpha} \ , && \text{if } C \geq C_0 \ . \end{aligned} \tag{12}$$

Such a CCDF is graphed in Fig. 2.

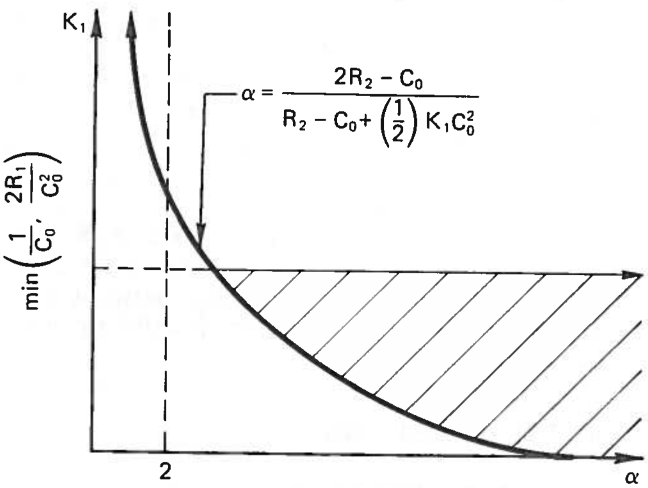

Fig. 1. Permissible values of α and K_1 for the case $C_0 \leq R_2$.

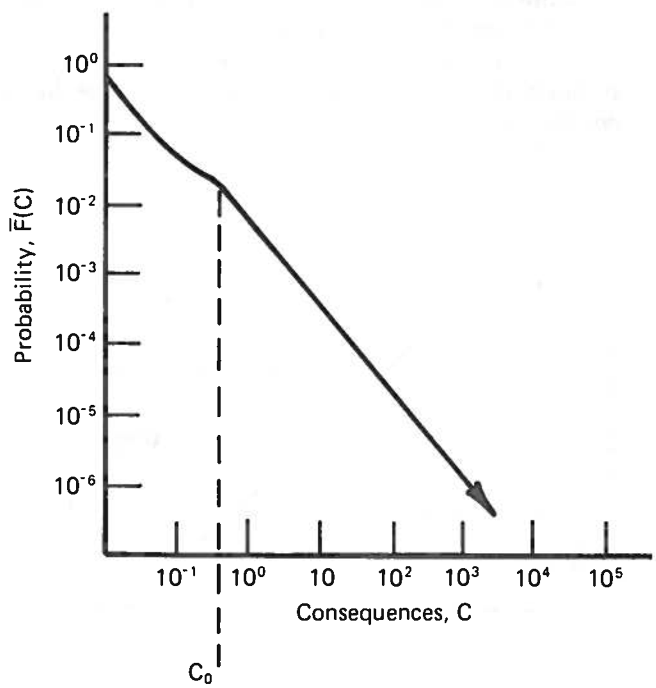

Fig. 2. The CCDF given by Eq. (12).

The total risk from the system is

$$\bar{R} = \int_0^\infty Cf(C)dC$$

$$= \frac{C_0}{\alpha - 2}\left[\alpha\left(1 - \frac{1}{2}K_1C_0\right) - 1\right] .$$

All of the concerns of items 1 through 4 are addressed by this formulation with the exception of item 2. However, when the CCDF of the system is compared to the ideal CCDF, accidents or categories of accidents posing unusually high relative risks will show up as perturbations in the smooth decrease of the CCDF, or as unexpected thresholds.

In this section, we have argued that a hypothetical CCDF is a suitable tool for the regulation of risk from a technological system. It provides for the imposition of quantitative risk standards, while remaining sufficiently flexible to allow the regulator to take into account several important aspects of risk. In the next section, we examine an alternative approach, the use of a limit line due to F. R. Farmer, and demonstrate why we consider the CCDF approach preferable. The distinction between the two approaches is also discussed, from a slightly different perspective, in Ref. 4.

III. THE FARMER LIMIT LINE

In 1967, Farmer[5] proposed a probability-consequence diagram and associated limit line for assessing, or limiting *a priori*, the risk to the public from nuclear reactor accidents. He expressed accident consequences in curies of ^{131}I released, and accident probabilities in terms of annual frequency.

The approach taken is illustrated in Fig. 3. Because of the logarithmic scales used, the line of *constant risk*

(Consequence) X (Frequency) = 1 Ci/yr

shown has slope −1 on the diagram. The *points* on the diagram represent individual accident sequences. Thus, point A corresponds to a low frequency, low consequence accident, point D to a high consequence, low probability accident, and so on. If the line shown in the diagram is chosen as the limit line, points above it have unacceptably high risk while points below it are adequately safe. The risk associated with a given accident may be reduced by taking measures to move the corresponding point on the diagram *down* (reduced probability) or *to the left* (lower consequences). For example, a new safeguards system might be introduced to reduce accident consequences.

Other limit lines, besides those of constant risk, may be used. For example, considerations of risk aversion may lead us to require that accident probabilities decrease at a faster rate than accident consequences increase. This requirement leads to limit lines of slope less than −1 on the probability-consequence diagram. Farmer himself proposed a line of slope −1.5, as shown in Fig. 4.

This limit line corresponds to a reduction in probability of three orders of magnitude for each two order of magnitude increase in consequences. The initial curved part of the limit line is drawn to control nuisance releases. We do not want even very small accidental releases to have a very high frequency. The choice of a line of slope −1.5, as well as

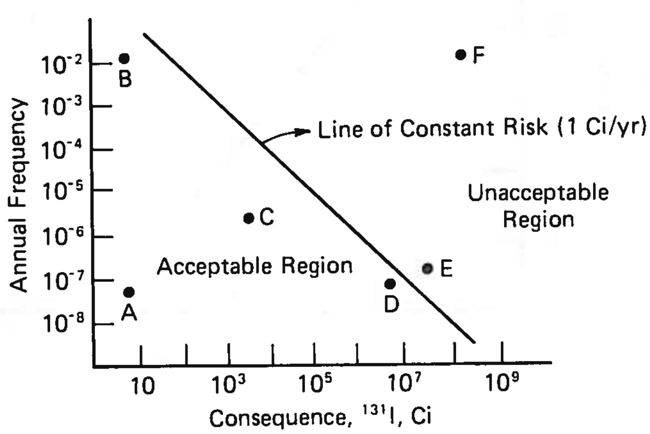

Fig. 3. The Farmer limit line.

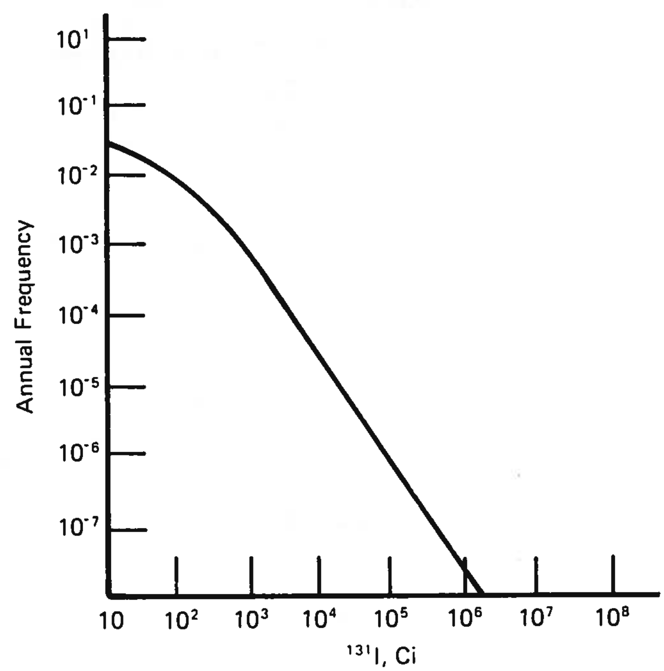

Fig. 4. Risk averse limit line.

the precise location of the line, is somewhat arbitrary. However, both parameters may be adjusted easily in accordance with the requirements or regulatory agencies or public opinion.

As described above, the Farmer limit line sets acceptability criteria only for the risk due to individual accident sequences. This creates an important limitation to the methodology. Namely, the Farmer limit line cannot be used to provide an estimate of, or limit to, overall system risk. As pointed out by Okrent,[6] for example, a large concentration of accident sequences near the limit line could lead to unacceptable overall risk, despite the adequately low risk posed by each individual sequence. In fact, many different values for overall risk are compatible with a particular placement of the limit line. Overall risk must be evaluated by the methods described in the Appendix.

Attempts to evaluate overall risk by integrating the Farmer limit line itself have lead to considerable confusion. An instructive example is furnished by the controversy surrounding a 1972 paper of Meleis and Erdmann.[7] Following Otway and Erdmann,[8] these authors suggested that a value of 10^{-7} per person-year be adopted as the upper limit for mortality risk incurred by a person living at the exclusion distance from a nuclear power plant. They then attempted to calculate the annual individual mortality risk to such a person represented by the placement of the "risk averse" limit line of Fig. 4. To simplify the integration, they approximated the curved part of the limit line by a straight line as shown in Fig. 5. The authors estimated the individual mortality risk from release of a single curie of ^{131}I as 0.682×10^{-6}. They then expressed the ^{131}I release C as a function of its annual probability P and wrote

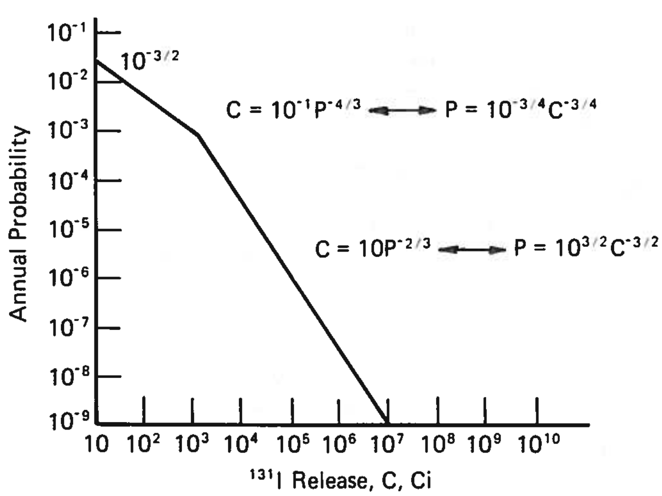

Fig. 5. Meleis-Erdmann limit line.

Individual mortality risk per year

$$= 0.682 \times 10^{-6} \left(\int_{10^{-9}}^{10^{-3/2}} C dP \right)$$

$$= 0.682 \times 10^{-6} \left(\int_{10^{-9}}^{10^{-3}} 10 P^{-2/3} dP \right.$$

$$\left. + \int_{10^{-3}}^{10^{-3/2}} 10^{-1} P^{-4/3} dP \right)$$

$$= 3.42 \times 10^{-6} \ . \tag{13}$$

In a Letter to *Nuclear Safety*, Farmer[9] criticized this method of calculation and proposed his own solution, based on the work of Beattie et al.[10] He interpreted the limit line as the PDF of consequences measured on a logarithmic scale (see the Appendix). He therefore, following Eqs. (A.8) and (A.6), calculated as follows:

Individual mortality risk per year

$$= 0.682 \times 10^{-6} \left(\int_{10}^{10^7} \frac{P}{\ln 10} dC \right)$$

$$= 0.682 \times 10^{-6} \times (\ln 10)^{-1}$$

$$\times \left(\int_{10}^{10^3} 10^{-3/4} C^{-3/4} dC + \int_{10^3}^{10^7} 10^{3/2} C^{-3/2} dC \right)$$

$$= 1.4 \times 10^{-6} \ . \tag{14}$$

Note that this answer differs from that given in the quoted Letter[9] because of other changes, not relevant to the present discussion, made by Farmer to the computations of Meleis and Erdmann.

In his reply to Farmer, Erdmann[11] claimed that his method of calculation should be the same as Farmer's. This is, however, not the case. In fact, Erdmann's method is tantamount to assuming that the limit line represents the CCDF of accident consequences. For, suppose $P = \bar{F}(C)$. Then, from Eqs. (A.5) and (A.6), we find

Individual mortality risk per year

$$= 0.682 \times 10^{-6} \left[\int_{10}^{10^7} - C \frac{d}{dC} \bar{F}(C) dC \right]$$

$$= 0.682 \times 10^{-6} \int_{10^{-9}}^{10^{-3/2}} C dP \ , \tag{15}$$

which is the Meleis-Erdmann calculation, on substituting $P = \bar{F}(C)$ in the first integral in Eq. (15).

Thus, the different values for annual mortality risk obtained by Farmer and Meleis-Erdmann are traceable to differing interpretations of the limit line. According to Farmer's original paper,[5] the placing of the limit line of Fig. 5 is intended to reflect an annual frequency of 10^{-3} for releases of 10^3 Ci. In a continuous model, the frequency of any particular release is taken as zero, so the 10^{-3} frequency must be interpreted as applying to some *range* of releases. This range could be taken, for example, as a decade interval about 10^3 Ci (i.e., $10^{2.5}$ to $10^{3.5}$ Ci), or as all releases of 10^3 Ci or more. This latter range fits well with the interpretation of the limit line as a CCDF. Neither range, however, agrees with an interpretation of the line as a logarithmic density function. This interpretation would yield, for example, from Eq. (A.8),

Prob (consequences of 10^3 Ci or more)

$$= \int_{10^3}^{10^7} f(C)dC$$

$$= \int_{10^3}^{10^7} \frac{10^{3/2}C^{-5/2}}{\ln 10} dC$$

$$= 2.9 \times 10^{-4} \quad . \qquad (16)$$

Thus, it appears that the Meleis-Erdmann interpretation of the limit line as a CCDF is in closer agreement with the intent of Farmer's paper. However, neither interpretation is consistent with the original definition of the line as a limit on the risk due to *individual* accident sequences.

To summarize, considerations of overall system risk are difficult to accommodate within the framework of the Farmer limit line. Because the CCDF approach avoids such difficulties while still allowing consideration of the other important aspects of risk discussed in Sec. II, we consider it superior to the Farmer approach.

IV. LIMIT LINES IN THE NUCLEAR LICENSING PROCESS

In this section, we examine some actual or proposed rules for the licensing and operation of nuclear power plants in Canada and the United Kingdom and their relation to the limit lines discussed previously.

IV.A. Proposed Canadian Criteria

Explicit probabilistic criteria have been employed in the licensing of nuclear power plants in Canada for at least 15 years. Over time, these criteria have become both more stringent and more detailed. As discussed in Ref. 12, a plant is considered to consist of the process system and the safety systems (protective system and containment system). The first safety criteria adopted are shown in Table I. Limits were also placed on total population dose for the two accident categories.

TABLE I

Original Probabilistic Safety Criteria for Canadian Nuclear Plants

	Maximum Permissible Annual Frequency	Maximum Permissible Dose to Any Member of Public [rem (whole body)]
Single failure Serious failure of process system	3×10^{-1}	0.5
Dual failure Failure of process system combined with failure of a safety system	1×10^{-3}	25

Though these criteria were made more stringent over the years, the maximum permissible annual frequency for dual failures being reduced to 3×10^{-4}, it was eventually recognized that more detailed rules were needed to allow for failures with differing rates of occurrence and consequences. The report[13] of a working group set up to study this problem proposed such rules, shown in Table II.

Events with annual frequency in the 10^{-7} to 10^{-6} range must be considered as contributing to the overall probability of the relevant dose interval, while events with annual frequency $<10^{-7}$ are considered incredible, though reasonable assurance must be provided that their overall probability is $<10^{-6}$.

Here, "dose" means: dose to an individual member of the public assumed located at the site boundary. Though the report recognizes the importance of "continually assessing the radiation effect on the population from all sources," no direct limits are placed on population dose since "for assessing design and operating safety the radiation received by an individual is a much more meaningful factor." The

TABLE II

Proposed Safety Criteria for Canadian Nuclear Plants

Dose Interval (rem)		Maximum Permissible Total Annual Frequency for Failures in Given Dose Interval
Whole Body	Thyroid	
0 to 0.05	0 to 0.5	10^{-1}
0.05 to 0.5	0.5 to 5	10^{-2}
0.5 to 5	5 to 50	10^{-3}
5 to 10	50 to 100	10^{-4}
10 to 30	100 to 300	10^{-5}
30 to 100	300 to 1000	10^{-6}

distinction between single and dual failures is retained in the report. However, it is treated principally as a useful way of organizing the risk assessment process. It is explicitly recognized that "it is only the probability of an event that should determine the acceptable consequences and not whether the event is a single or dual failure."

Table II presents limits on the sum of the probabilities of all accidents with consequences falling in prescribed consequence categories. The first three categories have constant risk equal to that associated with normal operation. The risk of the three high consequence categories is lower in recognition of the risk averse attitude of the public toward high consequence accidents. The philosophy here is clearly very close to that behind the Farmer limit line and the safety criterion proposed in Sec. II of this paper. It is instructive to plot Table II as a CCDF (see Fig. 6). The CCDF consists of two straight lines on a log-log plot, one with slope -1 corresponding to low consequence accidents and the other with slope -2 corresponding to more severe accidents. The placement of the lines is dictated by the requirement that the risk from each consequence category should not exceed that from normal operation.

In summary, then the proposed Canadian criteria are in close agreement with the limit-line ideas of Sec. II. Limits are placed on the risk due to defined consequence categories. The resulting CCDF, as a limit line on overall system risk, satisfies the criterion proposed in Sec. II for adequately fast decrease of probability with increasing consequence. This is because a CCDF of slope $-\alpha$ on a log-log plot corresponds to a PDF of slope $-\alpha - 1$ on a log-log plot [compare Eqs. (8) and (12)].

IV.B. U.K. Criteria

The approach to reactor licensing and safety in the United Kingdom[14-16] is based fundamentally on the principle that risks should be made "as low as reasonably achievable" (ALARA). Thus, all numerical criteria for safety are subordinate to the ALARA requirement and are not to be relied on exclusively. For example, if simple, inexpensive means are available to reduce a certain risk, then the licensee must do so, even if the present level of risk satisfies the appropriate numerical standard.

In recognition of the diversity of accident probabilities and consequences, numerical criteria have been adopted that limit the probability of any accident sequence in terms of the severity of its consequences. These criteria are shown in Table III, based on information in Refs. 14 and 15.

In agreement with the original idea of the Farmer limit line, and in contrast to the Canadian approach, explicit limits are not placed on the *overall* probability of various consequence categories. As discussed previously, this approach can potentially lead to unacceptable overall risk even when the risk of each individual accident sequence is acceptably low. Such a situation can occur if many sequences have risks close to the permissible upper limit. In recognition of this problem, it is required that the licensee demonstrate that "all reasonable steps

TABLE III

U.K. Licensing Standards

Annual Probability	Maximum Permissible Dose to Any Member of the Public (rem)
$>3 \times 10^{-2}$	1.7×10^{-2}
3×10^{-4} to 3×10^{-2}	5.0×10^{-1}
$<3 \times 10^{-4}$	1.0×10^{1}
Negligible	$>1.0 \times 10^{1}$

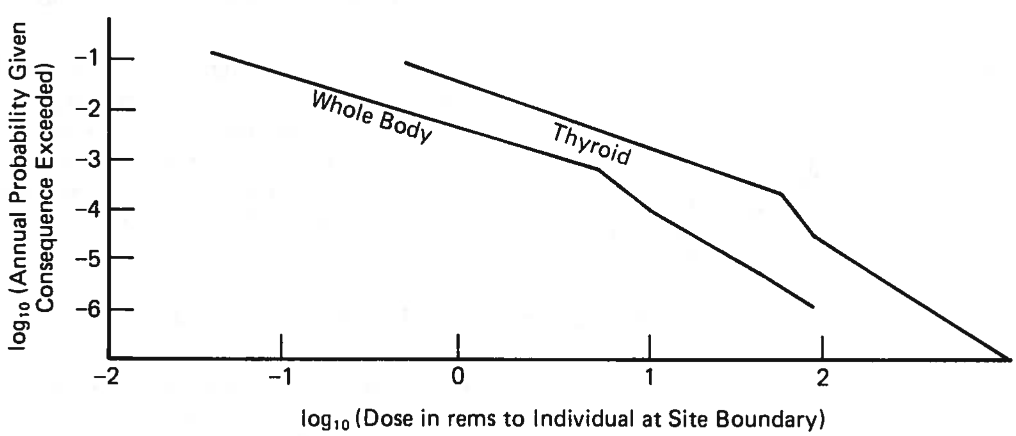

Fig. 6. Proposed Canadian licensing criteria.

have been taken in the design of the plant to avoid a distribution of faults having frequencies or consequences such that their cumulative effect on the overall risk would be significant" [Ref. 14, paragraph 28(iii)].

The criteria of Table III may be represented graphically (see Fig. 7). The term "negligible" in the table has been interpreted as 10^{-7}. The shaded area represents accident sequences whose risk is acceptable. The trend of the limit line may be assessed by means of a suitable least-squares fit. The least-squares line shown was fitted to the points in Table IV. These points are reasonably representative of Table III. The slope of the line is -1.52 on a log-log plot, as shown. The criteria of Table III, therefore, lead to a limit line very similar to Farmer's risk averse line though without an explicit cutoff on the probability of low consequence "nuisance releases."

TABLE IV

Representative Points from Table III

Annual Probability	Dose (rem)
3×10^{-1}	1.7×10^{-2}
3×10^{-3}	5×10^{-1}
1×10^{-5}	1×10^{1}
1×10^{-7}	1×10^{2}

V. CONCLUSIONS

In this paper, we have proposed a hypothetical CCDF as a suitable limit line for the regulation of risk from a technological system. The use of a CCDF allows the regulator to impose quantitative risk standards while, at the same time, taking into account several important aspects of risk, including overall risk, unusually high relative risks ("sore thumbs"), risk aversion, and high frequency, low consequence accidents.

We consider the CCDF approach preferable to its principal competitor, the Farmer limit-line approach, because the latter has difficulty accommodating considerations of overall risk. As shown in Sec. III, attempts to evaluate overall risk by integrating the Farmer line have been a source of considerable confusion in the literature. Both methods have been applied in practice to the licensing of nuclear power plants, the CCDF approach in Canada and the Farmer limit line in the United Kingdom, where it is augmented by a separate, qualitative standard for overall risk. The need for such a separate standard is, we believe, a drawback to the Farmer approach.

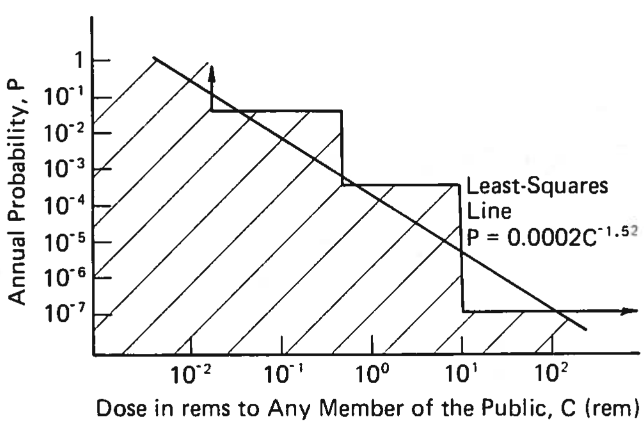

Fig. 7. Limit line for U.K. reactor licensing.

APPENDIX

PROBABILITY CONCEPTS

In this Appendix, we define and illustrate the various concepts of elementary probability used in this paper. We treat the discrete and continuous cases separately both for the sake of clarity and because the differences between the two cases have been a persistent source of confusion in the literature on limit lines. We assume that we have a fixed system in which various accidents can give rise to a spectrum of consequences, C, with associated probabilities of occurrence per year, P.

The Discrete Model

The totality of possible accidents $A_1, A_2, \ldots, A_n$ can give rise to consequences $C_1, C_2, \ldots, C_n$ with respective probabilities $P_1, P_2, \ldots, P_n$. Here

$$P_i \geqslant 0 \ , \quad i = 1, \ldots, n \ , \quad \text{and} \sum_{i=1}^{n} P_i = 1 \ .$$

Quite often, "accident" A_1 may in fact represent the category "no accidents in a year" and may have a probability close to unity. This situation obtains in the case where the system is a nuclear reactor, for example. The corresponding consequence $C_1 = 0$.

The sequence $P_1, P_2, \ldots, P_n$ is called the *probability function* of consequences. It may be represented graphically by a bar chart, as shown in Fig. 8.

The cumulative distribution function (CDF) of consequences is defined by

$$F(C) = \text{Prob (consequences less than or equal to } C)$$

$$= \sum_{C_i \leqslant C} P_i \ . \tag{A.1}$$

In the present discrete case, F is a step function as shown in Fig. 9 for the case $n = 5$. The CCDF of consequences is given by

$$\begin{aligned}\bar{F}(C) &= \text{Prob (consequences exceeding } C) \\ &= 1 - F(C) \ . \end{aligned} \tag{A.2}$$

The CCDF in the discrete case is also a step function as shown in Fig. 10.

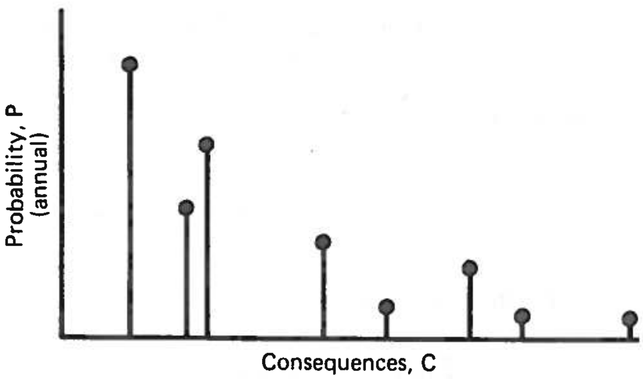

Fig. 8. Probability function of accident consequences.

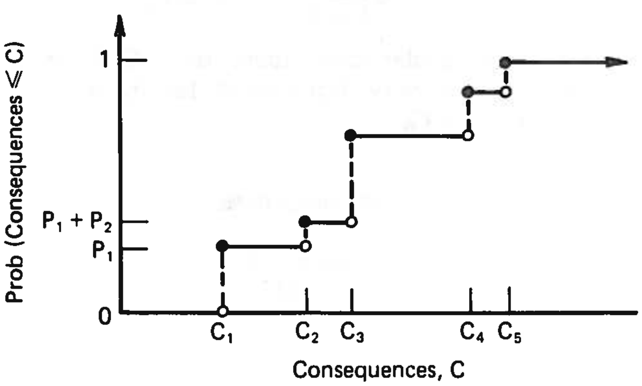

Fig. 9. The CDF of consequences.

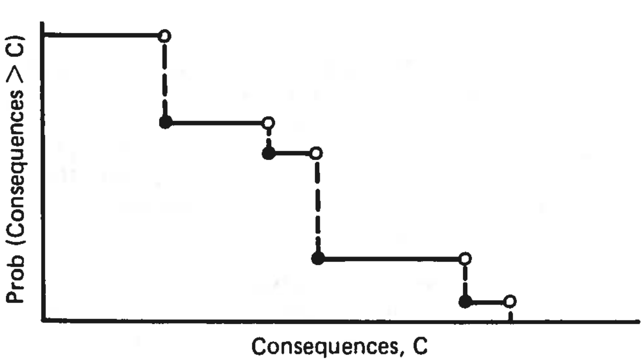

Fig. 10. The CCDF of consequences.

The *risk* associated with accident A_i is defined here as P_iC_i, the product of probability and consequence. The *overall system risk* is defined as

$$R = \sum_{i=1}^{n} P_iC_i \ . \tag{A.3}$$

The quantity R can also be determined from the CDF or the CCDF since the steps in these functions determine the probabilities P_i.

The Continuous Model

We assume here a continuous spectrum of consequences $0 \leqslant C < \infty$. This does not necessarily imply consideration of an infinite number of accidents. For example, a finite number of accidents each giving rise to a continuous spectrum of consequences could lead us to consider a continuous model. The continuous analog of the discrete probability function is the PDF $f(C)$ (see Fig. 11).

The probability of obtaining consequences between C and $C + dC$ is $\sim f(C)dC$ and so the probability of consequences between levels C_1 and C_2 is

$$\int_{C_1}^{C_2} f(C)dC \ .$$

The CDF and CCDF for the continuous case are given by

$$F(C) = \int_0^C f(x)dx$$

and

$$\begin{aligned}\bar{F}(C) &= 1 - F(C) \\ &= \int_C^{\infty} f(x)dx \ , \end{aligned} \tag{A.4}$$

and are illustrated in Figs. 12 and 13, respectively. The PDF can be recovered from the CDF or the CCDF using the formulas

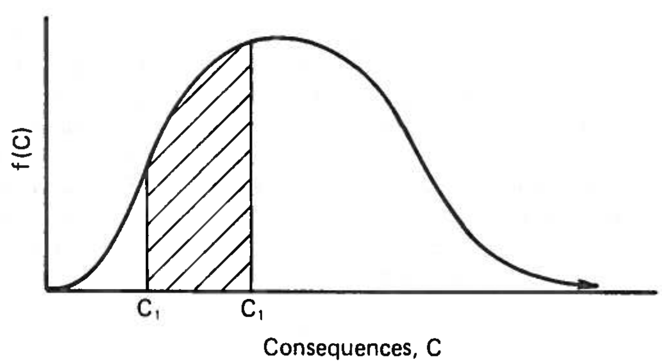

Fig. 11. The PDF of consequences.

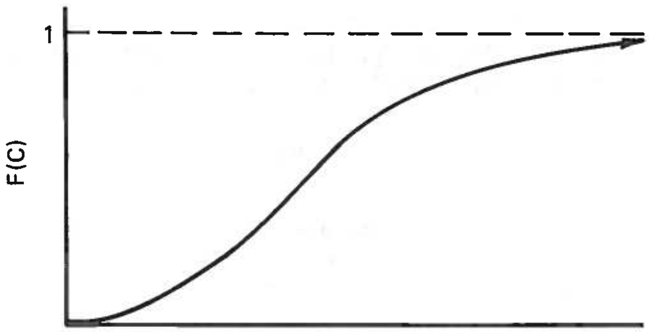

Fig. 12. The CDF for continuous case.

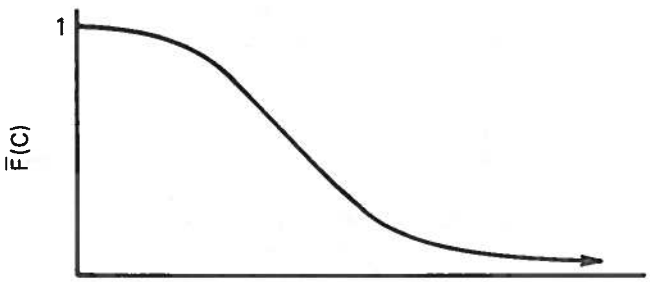

Fig. 13. The CCDF for continuous case.

$$f(C) = \frac{d}{dC} F(C)$$
$$= -\frac{d}{dC} \bar{F}(C) \ . \qquad (A.5)$$

The *pointwise risk* of the system is defined to be the function $Cf(C)$, by analogy with the discrete case, and the overall risk is given by

$$R = \int_0^{\infty} Cf(C) dC \ . \qquad (A.6)$$

Use of Logarithmic Scales

In many situations, order of magnitude differences are observed between the probabilities and/or consequences of different possible accidents. In such cases, the use of logarithmic scales for probabilities and/or consequences is appropriate. One may also express a continuous PDF in such scales. For example, Beattie et al.[10] employ a PDF $g(C)$ for consequences measured on a logarithmic scale. This function is defined by the equation

$$dP = g(C) d(\log_{10} C)$$
$$= \frac{g(C)}{C \ln 10} dC \ . \qquad (A.7)$$

Thus,

$$g(C) = (\ln 10) Cf(C) \ , \qquad (A.8)$$

where $f(C)$ is the usual probability density of consequences. Using Eq. (A.8), any required probabilities may be calculated. For example, consider the special case where $f(C)$ is proportional to a power of C. Specifically, let

$$f(C) = (\alpha - 1) C_0^{\alpha-1} C^{-\alpha} \ , \quad C \geqslant C_0$$
$$0 \ , \quad \text{elsewhere} \ , \qquad (A.9)$$

where $C_0 > 0$ and $\alpha > 1$. These restrictions are necessary to ensure that f has a finite integral, which may be normalized to 1, as in Eq. (A.9). In this case, we have

$$g(C) = (\ln 10)(\alpha - 1) C_0^{\alpha-1} C^{1-\alpha} \ , \quad C \geqslant C_0$$
$$0 \ , \quad \text{elsewhere} \ . \qquad (A.10)$$

Further, the CCDF is given by

$$\bar{F}(C) = \int_C^{+\infty} f(x) dx$$
$$= 1 \ , \qquad C \leqslant C_0$$
$$= \left(\frac{C}{C_0}\right)^{1-\alpha} \ , \quad C \geqslant C_0 \ .$$

In this particular case, then, the CCDF and the logarithmic density function differ by a constant factor for $C \geqslant C_0$.

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REFERENCES

1. J. S. WU-CHIEN and G. APOSTOLAKIS, "On Risk Aversion in Risk Acceptance Criteria," *Reliab. Eng.*, **2**, 45 (1978).

2. G. H. KINCHIN, "Assessment of Hazards in Engineering Work," *Proc. Inst. Civ. Eng.*, **Part 1**, **64**, 431 (1978).

3. G. H. KINCHIN, "Design Criteria, Concepts and Features Important to Safety and Licensing," *Proc. Int. Mtg. Fast Reactor Safety Technology*, Seattle, Washington, August 19-23, 1979, Vol. I, p. 1 (1979).

4. D. BENINSON and L. LINDELL, "Critical Views on the Application of Some Methods for Evaluating Accident Probabilities and Consequences," *Proc. Conf. Current Nuclear Power Plant Safety Issues*, Stockholm, International Atomic Energy Agency (1980).

5. F. R. FARMER, "Reactor Safety and Siting: A Proposed Risk Criterion," *Nucl. Saf.*, **8**, 539 (1967).

6. D. OKRENT, "On Limit-Line Curves in Risk Evaluation," Letter to the Editor, *Nucl. Technol.*, **27**, 304 (1975).

7. M. MELEIS and R. C. ERDMANN, "The Development of Reactor Siting Criteria Based upon Risk Probability," *Nucl. Safety*, **13**, 22 (1972).

8. H. J. OTWAY and R. C. ERDMANN, "Reactor Siting and Design from a Risk Viewpoint," *Nucl. Eng. Design*, **13**, 365 (1970).

9. F. R. FARMER, Letter to the Editor, *Nucl. Safety*, **13**, 362 (1972).

10. J. R. BEATTIE, G. D. BELL, and J. E. EDWARDS, "Methods for the Evaluation of Risk," AHSB(S) R 159, U.K. Atomic Energy Authority (1969).

11. R. C. ERDMANN, Letter to the Editor, *Nucl. Safety*, **13**, 365 (1972).

12. D. G. HURST and F. C. BOYD, "Reactor Licensing and Safety Requirements," Paper #72-CNA-102, Canadian Nuclear Association (1972).

13. "Proposed Safety Requirements for Licensing of CANDU Nuclear Power Plants," AECB-1149, Inter-Organizational Working Group, Atomic Energy Control Board (Canada) (1978).

14. "Safety Assessment Principles for Nuclear Power Reactors," HM Nuclear Installations Inspectorate (1979).

15. R. GAUSDEN, "The Application of Safety Criteria in the Assessment of Nuclear Power Plants in the UK," *J. Inst. Nucl. Eng.*, **19**, 106 (1978).

16. W. S. GRONOW and G. LEWIS, "Regulatory Control of Nuclear Power Stations in the United Kingdom," *J. Br. Nucl. Energy Soc.*, **2**, 7 (1979).